DETERMINATION OF PHOTOELASTIC CONSTANTS IN THE PRESENCE OF TILT OF THE AXES

K. V. KRISHNA RAO

PHYSICS DEPARTMENT, OSMANIA UNIVERSITY, HYDERABAD-7

(*Received January 2, 1961*)

ABSTRACT. This paper describes a direct method of determining the differential stress-optical constants, when the axes of polarisation in a stressed crystal do not coincide with the principal directions of stress. The method is verified by studies on barium nitrate and strontium nitrate crystals.

INTRODUCTION

With the application of the group-theoretical methods by Bhagavantam (1942) to derive the number of non-vanishing photoelastic constants for different classes of crystals and the discovery of several errors in the schemes given earlier by Pockels (1889), interest in the photoelastic effect in crystals was revived and an intensive study of the subject was undertakn by Bhagavantam and collaborators in recent years (Nye, 1957; Krishnan, 1958). During the course of these studies, it was found (Bhagavantam and Krishna Rao, 1953a) that, for some orientations of cubic crystals, the principal axes of polarisation of the stressed crystal do not coincide with the principal directions of stress. This phenomenon has been referred to as the tilt of the axes. When there is tilt of the axes, if the usual experimental method for determining the differential stress-optical constants, is employed, one should first find the positions of the axes of polarisation of the stressed crystal and adjust the Babinet compensator, such that its principal directions coincide with the axes of polarisation of the stressed crystal. On the other hand, if the principal directions of the compensator are kept vertical and horizontal, as usual, and the crystal is stressed vertically, it has been found that the shift of the Baninet fringe is not proportional to the applied stress and the fringe vibrates about the initial position, as the stress is gradually increased. It will now be shown that, from a knowledge of the stress required to bring back the Babinet fringe to its initial position, the stress-optional constant can be evaluated directly, without necessitating the determination of the tilt of the axes.

THEORY

Let OX_1, OY_1 (Fig. 1) be the principal directions of polarisation of the stressed crystal in the XY plane (normal to the direction of observation), OX_2, OY_2 the principal directions of the compensator and OP the direction of vibration of the incident plane polarised beam of light of amplitude a. Let α and β be angles X_1OP and X_1OX_2 respectively.

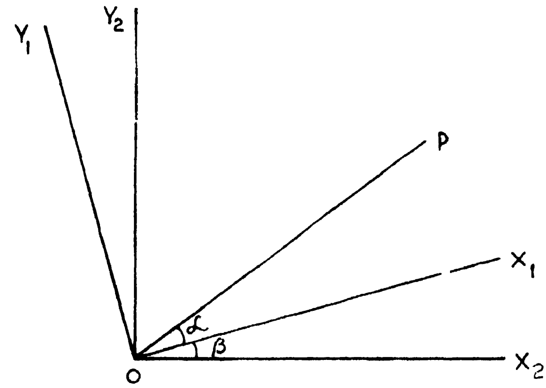

Fig. 1. Showing the principal directions of the stressed crystal (OX_1, OY_1), the principal directions of the compensator (OX_2, OY_2) and the direction of vabration (OP) of the incident light.

On entering the stressed crystal, the incident beam will be resolved into two components, one of amplitude $a \cos \alpha$ and direction of vibration OX_1 and the other of amplitude $a \sin \alpha$ and direction of vibration OY_1. These components will have a phase difference, say δ, when they leave the crystal. When the beam enters the compensator, each of the above-mentioned two components will split further into two components with their vibration directions along OX_2 and OY_2. The amplitude of the component, with the vibration direction along OX_2, is the resultant of the two components of amplitudes $a \cos \alpha \cos \beta$ and $-a \sin \alpha \sin \beta$ and phase difference δ. Similarly, the amplitude of the component with vibration direction along OY_2 is the resultant of two components of amplitudes $a \sin \alpha \cos \beta$ and $a \cos \alpha \sin \beta$ and phase difference δ. The amplitude A of the component with the vibration direction along OX_2 is given by

$$A^2 = a^2\{\cos^2(\alpha-\beta) - \tfrac{1}{2} \sin 2\alpha \sin 2\beta(1+\cos \delta)\}. \qquad (1)$$

The phase of this vibration Δ_1 is given by

$$\tan \Delta_1 = -\frac{\sin \alpha \sin \beta \sin \delta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta \cos \delta} \quad \ldots \quad (2)$$

Similarly, the amplitude B of the component whose vibration direction is along OY_2 is given by

$$B^2 = a^2 \{\sin^2 (\alpha+\beta) - \tfrac{1}{2} \sin 2\alpha \sin 2\beta(1- \cos \delta)\} \qquad (3)$$

and its phase Δ_2 is given by

$$\tan \Delta_2 = \frac{\sin \alpha \cos \beta \sin \delta}{\cos \alpha \sin \beta + \sin \alpha \cos \beta \cos \delta} \tag{4}$$

From Eqs. (2) and (4), we get

$$\tan (\Delta_2 - \Delta_1) = \frac{\sin 2\alpha \sin \delta}{\sin 2(\alpha+\beta) - 2 \cos 2\beta \sin 2\alpha \, . \sin^2 \delta/2} \tag{5}$$

The fringe shift in the compensator gives $(\Delta_2 - \Delta_1)$ which would obviously be equal to δ when the tilt of the axes β is zero. Eq. (5) shows that, as δ is increased, $(\Delta_2 - \Delta_1)$ first increases, reaches a maximum and then reduces to zero when $\delta = \pi$. On a further increase of δ, $\Delta_2 - \Delta_1$ changes sign, reaches a maximum and again reduces to zero when $\delta = 2\pi$. Thus the Babinet fringe completes one oscillation as the phase difference δ increases from zero to 2π. The variation of $\Delta_2 - \Delta_1$ with δ, evaluated for values of α and $(\alpha + \beta)$, 30° and 45° respectively, using Eq. (5), is shown in Fig. 2. It is clear that the strees P, required for one complete

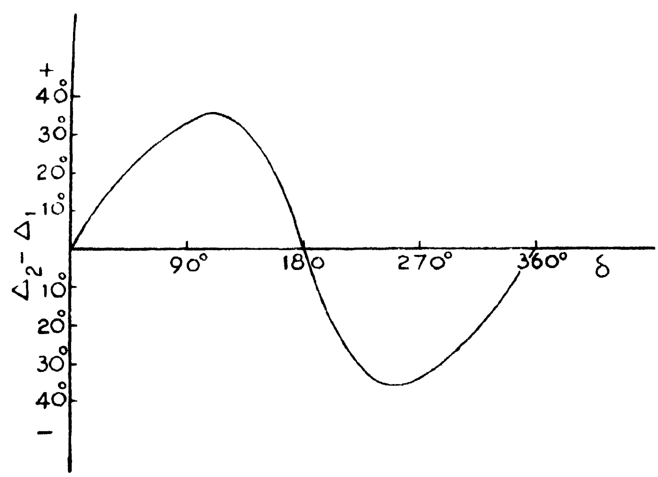

Fig. 2. Variation of $(\Delta_2 - \Delta_1)$ with δ.

oscillation of the Babinet fringe, gives the stress required for a path difference λ (wavelength of the light used) between the two components with their vibration directions along the principal directions of the stressed crystal. Hence the differential stress-optical coefficient C' is given by

$$C' = \frac{2\lambda}{n^3 P t},$$

where t is the thickness of the crystal parallel to the direction of observation and n the refractive index of the crystal in the unstressed state.

EXPERIMENTAL

To verify the foregoing method, crystal prisms of barium nitrate and strontium nitrate, with faces parallel to (111), $(01\bar{1})$ and $(\bar{2}11)$ planes, have been studied applying the stress, by a lever arrangement, along $[\bar{2}11]$ and making the observations along $[01\bar{1}]$ employing the usual arrangement (Fig. 3) for determining the differential stress-optical constants. The stress-optical constant C for this ori-

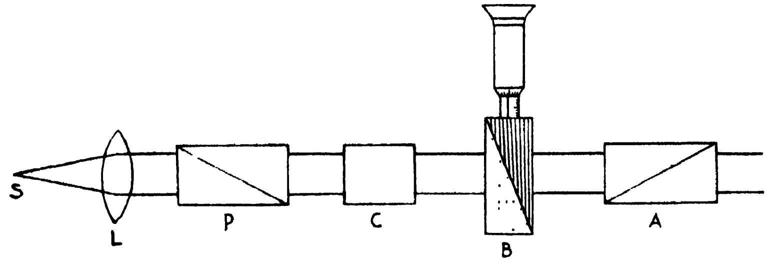

Fig. 3. Experimental set-up for determining differential stress-optical constants.
S—Source of light
L—Condensing lens
P—Polarising Nicol
C— Crystal
B—Babinet compensator
A–Analysing Nicol

entation of T_h (m_3) class of crystals, to which these two substances belong, is related to the stress-optical coefficients q_{11}, q_{12}, q_{13} and q_{44} by (Bhagavantam, 1953) :

$$C = \tfrac{1}{3}\sqrt{\tfrac{1}{4}\,(A+5q_{44})^2 + 2(A-q_{44})^2} \qquad \dots \quad (7)$$

where, $\quad A = \tfrac{1}{2}\,(2q_{11}-q_{12}-q_{13}).$

In the case of barium nitrate, the load required for one oscillation of the Babinet fringe is found to be 2380 grams, the mechanical advantage of the lever arrangement being 3.992. The length of the prism parallel to [111] direction, which enters the calculations, is 0.330 cm. With these values, taking n as 1.570 (Landolt and Bornstein, 1931), the stress-optical constant is evaluated, using Eq (6). The value obtained, 10.8×10^{-13} cm^2 dyne^{-1}, is found to be in agreement with 10.2, evaluated using Eq. (7), taking the values of A and q_{44} (Bhagavantam and Krishna Rao, 1953b) as 20.60 and 1.69 respectively.

For strontium nitrate, the load for one oscillation of the Babinet fringe is 3300 grams. The length of the prism parallel to [111] is 0.264 cm. The stress-optical constant, evaluated using Eq. (6), taking n as 1.567 (Landolt and Bornstein, 1931) is found to be 6.3, in close agreement with 6.27, evaluated using Eq (7), taking A and q_{44} ((Bhagavantam and Krishna Rao, 1954) as 13.61 and 1.38 respectively.

REFERENCES

Bhagavantam, S., 1942, *Proc. Ind. Acad. Sci.*, **A16**, 359.
Bhagavantam, S., 1953, *Proc. Ind. Acad. Sci.*, **A37**, 585.
Bhagavantam, S., and Krishna Rao, K. V., 1953a, *Proc. Ind. Acad. Sci.*, **A37**, 589.
Bhagavantam, S., and Krishana Rao, K. V., 1953b, *Acta Cryst.*, **6**, 799.
Bhagavantam, S., and Krishna Rao, K. V., 1954, *Curr. Sci.*, **23**, 257.
Krishnan, R. S., 1958, Progress in Crystal Physics, Madras.
Landolt, H. H. and Bornstein, R., 1931, Physikalische Chemische Tabellen.
Nye, J. F., 1957, Physical Properties of Crystals, Oxford : Clarendon Press.
Pockels, F., 1889, *Ann. der. Phys.*, **37**, 144.